



Knowing and Valuing Every Learner:

Culturally Responsive Mathematics Teaching

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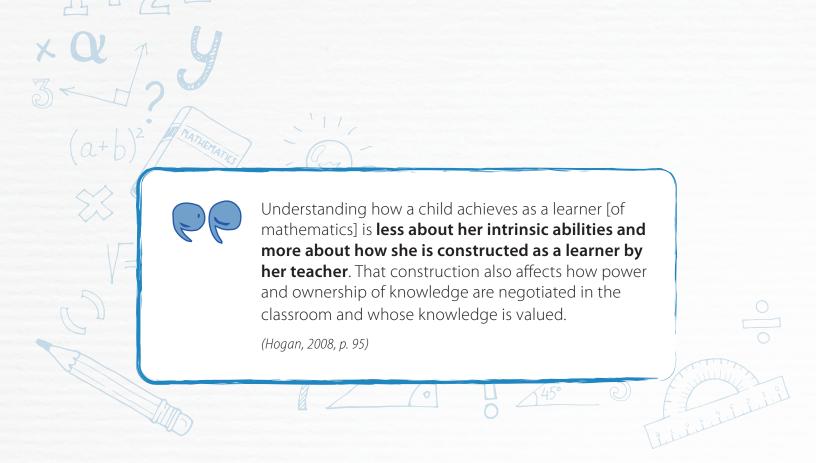
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Mark is a professor of Education at California State University, Fullerton and a highly regarded scholar and teacher of K–12 mathematics education. With over 40 publications to his credit, Mark has served on the National Council of Teachers of Mathematics (NCTM) Board of Directors and Executive Committee and recently published a book about creating productive mathematics learning environments for elementary students. He is best known for his collaborative work with educators in developing strategies that help students understand concepts, supporting instruction, and addressing issues of equity in mathematics education.



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Introduction: Defining Culturally Responsive Mathematics Teaching

We know from decades of research that:

- Student success with mathematics is primarily due to opportunities to learn meaningful mathematics—and not due to innate intelligence;
- effective mathematics teaching cultivates the mathematics abilities of all students;
- equitable access and support in learning mathematics includes attention to students' reasoning and identities (Boaler & Staples, 2008; Gutiérrez, 2013; Kisker, et al., 2012; Malloy & Malloy, 1998; NCTM, 2014; National Research Council, 2009; Razfar, Licón Khisty, & Chval, 2009).

The idea of **culturally responsive mathematics teaching (CRMT)** is premised on creating a learning environment focused on mathematical sense making in which each student feels valued for who they are, for their ways of engaging in mathematical reasoning, and for their contributions to the collective success of those within the classroom community (*Aguirre & Zavala, 2013; Ladson-Billings, 1995; 2009; 2014*).

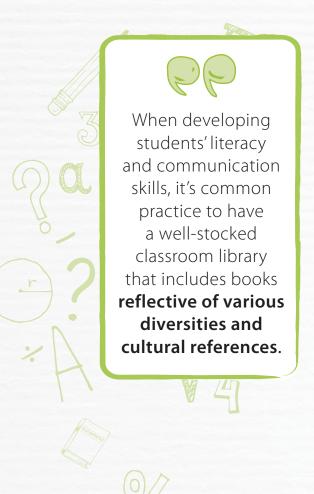
Much more than inserting students' names and cultural referents into mathematics problems, CRMT requires teachers to (re)consider the mathematics learning environment they've created and how well it reflects these four elements:

- Supporting deep learning: How to ensure student success with coherent and connected mathematical understandings
- **Engaging and valuing identities:** How to honor students' experiences, communication practices, and communities
- 3. Sharing authority: How to build inclusive, collaborative norms and routines
- 4. Applying mathematics: How to use mathematics to understand and investigate meaningful situations

CRMT is not just about making mathematics tasks relevant to students (although this is a good first step). It is about creating a community of mathematics learners who value collaboration and see mathematics as a way of reasoning with and about quantities. It is about inviting all students into mathematics as competent participants whose ways of thinking and reasoning are worth sharing, discussing, and refining. Most of all, it is about ensuring each and every learner not only has success with mathematics but also comes to see mathematics as part of their identity and as a tool for examining their world through a quantitative lens. Aguirre and Zavala (2013, p. 6) offer the following explanation:

. . . culturally responsive mathematics teachers leverage mathematical learning by expanding children's mathematical thinking, building bridges between previous knowledge and new knowledge, supporting bilingualism and academic language development, fostering connections with cultural funds of knowledge and experiences, and cultivating critical mathematical knowledge that enables students to analyze and address authentic problems.

The idea of using students' cultures, identities, and familiar contexts to connect them with academic content is nothing new in other content areas. When developing students' literacy and communication skills, it's common practice to have a wellstocked classroom library that includes books reflective of various diversities and cultural references. (Moll & Gonzalez, 1997; Newman, Arthur, Staples, & Woodrow, 2016; Turner & Danridge, 2014; Turner & Kim, 2005; Wheeler & Swords, 2004). These ideas resonate with teachers due to an obvious connection between children's ways of communicating and the goal of developing academic literacy. However, learning mathematics should be seen in a similar way; students bring with them ways of thinking and reasoning with and about quantities and quantitative contexts as well as ways of interacting and communicating—that serve as a foundation upon which to build their understanding of mathematics.



Some ideas for reflecting on each of the four elements of CRMT are shared on the following pages. Also included are examples from teachers of mathematics putting CRMT into practice and a list of resources to further your learning about CRMT. It is important to recognize that adopting a CRMT approach does not mean having to rewrite your entire curriculum. You can use a strong set of standard instructional materials, such as *Ready Classroom Mathematics*, and think about how to make the best of use of these within your classroom context. **CRMT is more about considering how to engage all students in meaningful learning**; while this may include modifications to lessons it also includes thinking about the norms and routines you establish, connecting parents and community to students' mathematics learning, and supporting every learner in developing a sense of ownership of mathematics.



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Nora's Story: The Impact of the **Learning Environment**

[This is a synopsis of research done by Maureen Hogan.]

The learning environment established within the classroom impacts greatly what students (and which students) are able to learn. The case of a young learner named Nora, a pseudonym, comes from research done by Maureen Hogan (2008) in a middle school within an Alaska Native Yup'ik community.

> **Nora began sixth grade** as a shy, reticent learner who had not yet found her voice in the mathematics classroom. Her teacher created a learning environment in which collaboration, communication, and justification were central to the aim of having students take ownership of mathematical ideas that emerged from their thinking. Students regularly shared their ideas, listened to the ideas of peers, and used these to make sense of mathematics concepts and relationships. The teacher supplemented the regular curriculum with contextual problems that were designed to connect key mathematical concepts, relationships, and skills with local cultural practices and ways of thinking.

One such unit, Building a Fish Rack, had students create racks to dry salmon, a common activity within their Alaska Native Yup'ik community, using methods based on relative units of measure (e.g., each rack is made proportional to the person who will use it). Students developed the concept of proportional reasoning through the Building a Fish Rack activity and then applied this to contexts involving Western standard units of measure (e.g., feet, meters). Nora especially enjoyed this unit as she had spent much of her time with her grandfather fishing and drying salmon.

During the course of the school year, Nora developed confidence in sharing her mathematical reasoning and emerged as a leader of class discussions about mathematics. Her teacher described Nora as a genius due to her often novel—and mathematically rich—ways of representing concepts and relationships. The learning environment that was created allowed Nora to connect her own experiences and interests with important mathematical ideas in ways that validated her identity and her community while at the same time supporting her academic success.

Nora's sixth grade mathematics teacher created an environment where learning mathematics was supported by students' collaborative engagement in processes of reasoning and sense making. This helped them take ownership of key concepts and relationships. In addition to a standard curriculum, the teacher used supplemental materials designed to connect local ways of reasoning quantitatively with formal mathematics standards. Nora thrived in sixth grade mathematics, being described as a genius.



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In seventh grade Nora had a different teacher who set up a learning environment focused on students individually following rote procedures to generate correct answers to decontextualized mathematics problems, earning rewards for catching peers with wrong answers, and being allowed to play games on the computer when work was completed.

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Nora did her best to follow the teacher's rules for "doing mathematics" but had trouble understanding why the steps were being done. She was reluctant to share her answers for fear of being caught with a mistake. And as time went on, she adopted the practice of many of her classmates—rushing mindlessly through her work so she could get to play games.

Her seventh grade teacher described Nora as working hard, but struggling with a D+ grade. When asked toward the end of the year about her experiences with seventh grade mathematics, Nora explained, "... the teacher only lets you learn some stuff" (Hogan 2008, p. 107).

In seventh grade, Nora's mathematics teacher created a learning environment focused on individually following rules to get answers that were evaluated as right or wrong. Nora struggled in seventh grade mathematics, earning a D+. Reflecting on the precipitous decline in Nora's engagement in and success with mathematics from sixth to seventh grade, Hogan concluded, "Nora simply performed the roles and followed the scripts available to her in these two different contexts. The roles and scripts in [seventh grade] were more limited than the creative, conceptual, and culturally meaningful ones available in [sixth grade]" (p. 109).

The learning environment that was created did not value nor build upon Nora's ways of thinking and interacting, did not reflect Nora's sense of identity or her community, and had the effect of marginalizing her academically. Within the course of one school year, Nora went from thriving to withering as a learner of mathematics. But it was not the student who had truly changed; it was the learning environment.

In other words, the learning environment in Nora's sixth grade classroom reflected the characteristics of CRMT while that of her seventh grade mathematics class simply did not value Nora's ways of interacting, thinking, and reasoning. As a result, her seventh grade mathematics norms left her with limited opportunities and access to meaningful mathematics. Nora's case powerfully illustrates the importance of using CRMT to create learning environments that reflect students' identities, experiences, and cultures in order to support them in taking ownership of important mathematical knowledge.



1. Supporting deep learning: How to ensure student success with coherent and connected mathematical understandings

Mathematics is at the core of CRMT, and it's important to remember that the learning environment supports students' success in mathematics. A focus on being culturally relevant in the absence of meaningful mathematics only exacerbates existing inequities in academic outcomes. While this is not unique to CRMT, there is an equity focus here in that for too many students, mathematics learning has been limited to rote skills and procedures, so-called "basic skills." The culturally responsive mathematics teacher recognizes the potential in every student to engage in mathematical thinking and finds ways to elicit students' mathematical reasoning.

Current standards recognize the need for students to understand the concepts behind the calculations and emphasize students' productive engagement in mathematical practices, such as explanation and justification of reasoning. Teachers must design and implement lessons that offer access and support to all students to meet the expectations of these more coherent and rigorous standards. This might look like starting a lesson with a Number Talk that invites students to share their strategies about a specific problem. Maybe it's using an instructional routine to engage students in discussing and connecting multiple representations used to solve contextual problems. Or, perhaps students might gather in front of a class Math Wall to reflect on their prior work and consider how it is related to a new question posed by the teacher (see *Yeh*, *Ellis*, & *Hurtado*, *2016* for these and other ideas).

Equally important is that teachers allow students to engage with multiple representations of mathematical concepts and relationships, comparing and contrasting these as a means to deepen understanding.



The culturally responsive mathematics teacher recognizes the potential in every student to engage in mathematical thinking and finds ways to elicit students' mathematical

reasoning.







For instance, one fifth grade teacher I worked with found that her students were struggling with the meaning of STRATEGY decimal numbers. Looking over her curriculum materials,

she realized that several representations of decimals were provided but separate from one another. She designed a lesson that instead required students to explore four representations of decimals (verbal, numerical, base-ten blocks, and coins), looking for and explaining connections among them. This gave her students opportunities to refine and extend their understanding of decimal numbers.

Ouestions to consider:

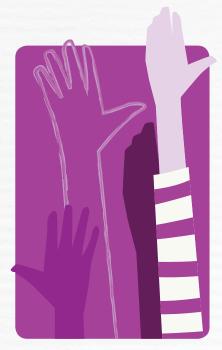
- What are the "big ideas" of mathematics my students will learn this unit? This year?
- What is the prior mathematical knowledge my students may need to draw on to make sense of these big ideas?
- How am I giving students time to develop their own mathematical thinking? And how am I communicating that their thinking is what matters, not just their answers?
- How am I ensuring my students learn mathematics with coherence? What connections and relationships will help them to build conceptual understanding?
- Am I encouraging students to make connections between multiple representations and strategies?



Equally important is that teachers allow students to engage with multiple representations of mathematical concepts and relationships,

comparing and contrasting these as a means to deepen understanding.





2. Engaging and valuing identities: How to honor students' experiences, communication practices, and communities

An important aspect of CRMT is to positively reflect students' cultures and communities within their learning in a nontrivial way. A group of sixth grade teachers that served a primarily Latino/a community brainstormed ways to more authentically engage parents in their children's learning.



One strategy they developed was to have students conduct short two-minute videos of interviews with a parent/ STRATEGY guardian during back-to-school night asking the simple question, "How do you use math in your job or everyday life?" The result was a trove of examples of applications of mathematics, many of which were easily connected to topics the students would be studying.

For example, one parent worked at a shipping company and described having to consider package dimensions and weights when stacking boxes in the delivery trucks. The teachers built a lesson around this video that asked students to figure out ways to package 24 boxes that measured one foot on each side. The parent's video interview served as an introduction to the idea of surface area and volume in real life and did well to hook students into the problem. You can find examples of these parent videos on the project website: Tacib.Weebly.com.

Communication is a part of everyone's daily life and must be fostered within the mathematics classroom. What CRMT asks us to do is recognize the ways our students communicate and allow them to use familiar, comfortable forms of communication as part of the learning of mathematics. This does not preclude the need to teach students academic language and mathematical notation, but the formal communication is best built after students have had opportunities to informally share ideas.

Research by Razfar, Licón Khisty, & Chval (2011) focused on one fifth grade teacher's success with supporting emerging bilingual students in developing proficiency in mathematics. What she did was provide students with multiple means for communicating their mathematical thinking—language, visuals, gestures, and even



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calculator keystrokes—and then used these to introduce more formal academic language and mathematical notation. In this way, students were allowed to participate fully, were recognized for their ways of thinking, and were provided scaffolded support in learning mathematical terminology and notation.

Students are part of communities outside of school, and these environments are rich spaces for seeing and applying mathematical ideas. Two strategies to gain knowledge about students and their communities are community resource maps (Reyes et. al., 2016) and home visits (Moll & Gonzalez, 1997; see also Teacher Visits.org). Designed to bring educators into the student spaces outside of school, these strategies also allow you to get a better sense of the ways students' lives are mathematically rich.

STRATEGY

For example, a fourth grade teacher took this a step further and asked students to take photos of mathematics in their communities and had them write problems based on these.

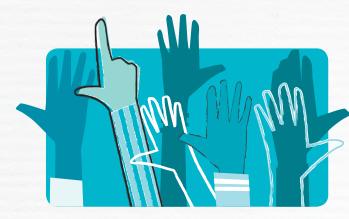
The teacher asked students to share their photos. One student took a picture of a fruit vendor and wrote a problem about the cost of ordering three cups of mango with chili and two cups of pineapple.



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Questions to consider:

- What have I learned about my students' prior knowledge and skills—including the languages they speak and ways they communicate—and in what ways do I encourage them to use these in their learning of mathematics?
- How do I learn about the experiences and interests of my students? And how do I communicate that these experiences and interests are valued in relation to their learning of mathematics?
- In what ways do students see themselves and their communities reflected in positive ways while learning mathematics?
- How do I invite authentic, meaningful parent/community engagement in my students' learning of mathematics?



Sharing authority: How to build inclusive, collaborative norms and routines

What are the ways that you allow students to be authors of mathematical knowledge? At its heart, mathematics is a human invention designed to allow for quantitative reasoning, both concrete and abstract. How is mathematical authority distributed in your classroom? What are ways you can allow for students—each and every one—to feel they have the power to be authors of the mathematics being learned? A classroom Math Wall is one way to provide a space to display students' reasoning about specific problems to be referred to later. This is also an area where technology can be used to great effect. Using tools such as Educreations™ and Google Classroom, students can share written work and recordings of their mathematical reasoning with peers, teachers, and parents.

The norms and routines that guide students' regular interactions with one another about mathematics must be intentionally developed to support a sense of shared responsibility and ownership of mathematics knowledge and skills.

Educreations™ is a proprietary mark of Educreations, Inc.

One example from Yeh, Ellis, & Hurtado (2016) involves recording examples of students productively engaged in specific activities (e.g,. pair discussion) and having other students watch short clips from these as they define class norms for interactions during mathematics. This gives a concrete referent for students' ideas and demonstrates your expectations for how students will interact. Role-playing is another strategy to establish norms for both acceptable and unacceptable behavior. Without such explicit attention to norms, there is a high likelihood for inequitable participation, favoring those who catch on to implicit norms. Likewise, setting up clear routines for engaging with mathematics tasks, such as those articulated by Kelemanik, Lucenta, & Creighton (2016; also see FosteringMathPractices.com/RoutinesForReasoning), provides structured ways for students to productively engage with mathematics and with one another. The routines in your classroom are essential to helping students learn that mathematics involves specific practices that develop skill with quantitative reasoning.

Questions to consider:

- In what ways do I position my students as having authority over the mathematics knowledge being developed?
- What norms have I established to ensure equitable participation and the reduction of unequal status among students?
- How do I support the different language needs in my classroom?
- What instructional routines do I use to scaffold students' engagement in productive mathematical discourse and collaboration?
- Who is given mathematical authority in my classroom? Who is not?
- What evidence do I have that my students are taking ownership of mathematics?





Applying mathematics: How to use mathematics to understand and investigate meaningful situations

While understanding mathematical concepts and relationships well enough to demonstrate proficiency with state-mandated assessments is important, it is even more critical for students to learn to use mathematics as a tool for investigating and critiquing issues within their communities. Too often "school mathematics" becomes compartmentalized and seen as something relevant only for an assessment or grade. What we want to work toward with CRMT is having students see mathematics as a tool for analyzing the world in which they live. This will strengthen students' interest and engagement in mathematics.

For example, in a lesson about linear equations, teachers have substituted examples from their textbook with examples from their students' community (e.g., the cost of going to the local skateboard park; the cost per ride at a local festival) and included photos and video clips, some in language(s) other than English, that are familiar to their to students. To assess understanding, students can be asked to create their own linear relationship problem using a context familiar to them and, when possible, include photos or short video clips.

A group of teachers in southern California collaborated to design an eighth grade unit about the Pythagorean Theorem. The lesson focused on identifying possible locations for a new cell phone tower near their school site and addressed a common complaint from students about poor cell phone reception in their area. The culminating assignment was a letter written to the school administration arguing for or against the idea of placing a cell phone tower on or near their campus. This provided students with an example of how mathematical problem solving could be combined with written communication to solve a real-world problem. A description of the unit and student work examples can be found here: *Tacib.Weebly.com*.



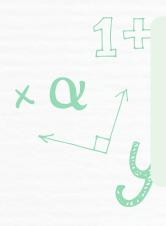
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One more example of how teachers can connect learning to student experiences is found in Turner and Strawhun's (2007) "Posing Problems That Matter: Investigating School Overcrowding." The authors describe a lesson involving proportional reasoning in a sixth grade urban New York City classroom drawn from students' concerns that girls at their school did not have sufficient bathroom capacity to accommodate their enrollment. Students were motivated to learn about concepts of ratio and proportion and learned how to present their findings to the local school board.

In each of these examples, there is a significant mathematics content objective as well as a significant social/cultural context; students were learning to make sense of mathematics and to use mathematics to make sense of some aspect of their lived experience. Common to each of these is a focus on meaningful mathematics content, relevant contexts, and opportunities for students to share their mathematical thinking. And in some cases, there were action steps that allowed students to take mathematics outside of their classroom and into their local community in ways that advocated for specific, mathematically supported changes in policy.



Questions to consider:

- How do I integrate concepts into instruction that are more relevant to my students?
- How am I helping my students to see ways to use mathematics to analyze and address issues within their communities?



Conclusion

Culturally Responsive Mathematics Teaching aims to allow students to take ownership of mathematical knowledge in ways that empower them to see themselves in mathematics and to use mathematics to examine real-world problems, especially those that are relevant in their communities. While there is no one "right" way to bring this approach to the classroom, the ideas shared within this paper are offered as examples, not exemplars, of what this could look like in practice.

It is important to note that not every lesson must include explicit cultural or community connections. Rather, by making it a habit to consider connections between one's students and the mathematics being studied, teachers are more likely to create an inclusive classroom learning environment in which all students, particularly those often marginalized by mainstream homogenized practices, feel a sense of belonging (*Byrd*, 2016). Learning to select and implement tasks that support high levels of cognitive demand and deep learning and developing a classroom culture that values students' mathematical thinking is just as important as the social and cultural elements.

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